Swansea University
Mathematics scholarship exam 2026

2 hours 30 minutes

Please attempt all the questions in section A, and then at most four from section B. Explanations of your solutions will form part of the assessment.

Section A

1. Write the repeating decimal expansion $2\cdot(77)^\bullet$ as a fraction with integer numerator and denominator (e.g. something like 57/13). Write the fraction 9/13 as a repeating decimal expansion.

2. Determine whether 653 is prime or not. Show your reasoning.

3. Solve the following simultaneous equations for x and y:

$$x - 2y = 2 \ , \quad 2x + y = 2 \ .$$

4. The number of bacteria in a culture increases by 6% every minute. If there were initially two million bacteria in the culture, how many would there be after 400 seconds?

5. Evaluate the integral

$$\int_0^2 \frac{1}{x+7}\,\mathrm{d}x \ .$$

6. There is one real number x which satisfies $x^4 = 1 - x^2$ and $0 \le x \le 1$. Calculate x to within an error of $\frac{1}{16}$.

7. Find the differential with respect to x of

$$\frac{e^x - 1}{e^x + 1} \ .$$

8. Evaluate the integral

$$\int_0^1 e^{2x}\,\mathrm{d}x \ .$$

9. A triangle has sides 4, 6 and 8. What is the size of the angle opposite the side of length 4?

10. Given that $x = 3$ is a root of the cubic $x^3 - 4\,x^2 + 2\,x + 3$, find the other two roots.

Section B

1. The function $f(x)$ is defined for real $x \neq 0$ by the formula

$$f(x) = 2\,x + \frac{1}{x}\ .$$

(a) For which real values y is there a solution to the equation $f(x) = y$?

(b) If $x \geq 1$, show that $f(x+1) \geq f(x) + 1$.

(c) For which real values x do we have $f(x) > x$?

2. Alice and Bob each have a cannon located on a large flat field. The cannons are separated by 10m and are pointed towards each other. Each cannon is pointed at an elevation of α radians to the horizontal. Alice fires her cannon towards Bob. Bob panics and fires his back at Alice exactly 1 second later. Given that both cannons fire balls at a speed of u m/s, show that the balls will collide in mid air (and therefore no-one will be hurt) if,

$$u = \sqrt{\frac{10g}{\sin(2\alpha)}},$$

and α is such that,

$$2u\sin\alpha > g.$$

where g is the acceleration due to gravity.

3.

a) State the principle of induction (any version will do).

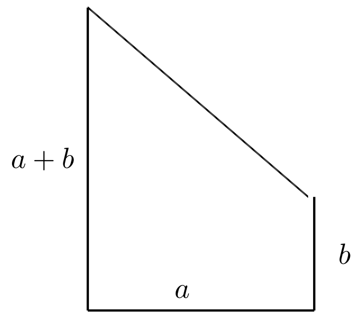
b) The following sum is given by a formula for $n \geq 0$, for some constants A and B:

$$\sum_{m=0}^{n} (4\,m + 3) \;=\; 2\,n^2 + A\,n + B\ .$$

Find the constants A and B.

c) Prove by induction that the formula, with your given values of A and B, gives the correct value for the sum for all $n \geq 0$.

4. A metal plate has four sides, one (length a metres) horizontal, two (length b and $a+b$ metres) vertical, and the fourth at 45° to the others.



a) What is the area of the plate? What is the length of the perimeter of the plate?

b) The length of the perimeter of the plate is fixed at 2 metres. Find a formula for b in terms of a which ensures that the length of the perimeter has this value.

c) Given that the length of the perimeter of the plate is fixed at 2 metres, find the value of a which maximises the area of the plate, and the maximum area of the plate.

5. A particle of mass m rests on a rough plane which is inclined at an angle θ to the horizontal. The coefficient of friction between the plane and the particle is μ. Consider the following three scenarios:

- If a horizontal force of magnitude P is applied to the particle then it is just sufficient to prevent the particle from sliding down the slope.
- If a force of magnitude $2P$ is applied parallel to the slope and acting up the slope, then the particle is just on the point of slipping up the slope.
- If instead a particle of mass $2m$ sits on the slope then a force $2P$ up the slope is just sufficient to prevent the particle from sliding down.

Deduce from this information that $\theta = \frac{\pi}{3}$ and $\mu = \frac{1}{\sqrt{3}}$.

6. Inspector Code of the Swansea police is investigating a robbery. The suspects are A, B, C and D, and it is known that nobody else could have been involved. The following facts have been established:

i) At least two people committed the robbery.

ii) If A was involved, then exactly three people did the robbery.

iii) If C was involved, then also D is guilty.

iv) If exactly two people were involved, then one of them was B.

Are the following statements definitely true, definitely false, or is it not known whether they are true or false?

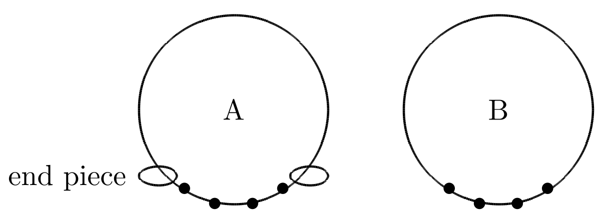
(a) All four suspects are guilty.

(b) If exactly two people were involved, then D is guilty.

(c) If exactly three people were involved, then A is guilty.

(d) D is guilty.

7. A company makes two sorts of necklaces. Design A consists of a string of stones between two gold end pieces. Design B has no end pieces, so the wearer can move the stones round the string to start with any of the stones. All stones of a particular type are effectively identical. The following picture shows the designs, with four stones.



(a) How many different necklaces can be made of type A with four stones, one each of the gemstones diamond, ruby, emerald and onyx?

(b) How many different necklaces can be made of type B with four stones, one each of the gemstones diamond, ruby, emerald and onyx?

(c) How many different necklaces can be made of type A with five stones, one each of the gemstones diamond, ruby, emerald and two onyx?

(d) How many different necklaces can be made of type B with five stones, one each of the gemstones diamond, ruby, emerald and two onyx?

(e) How many different necklaces can be made of type A with $n + 2$ stones, one each of the gemstones diamond and ruby, and n onyx?

(f) How many different necklaces can be made of type B with $n + 2$ stones, one each of the gemstones diamond and ruby, and n onyx?

8. A canon ball and a feather are dropped from the top of the leaning tower of Pisa which is 56m tall. The canon ball has mass M Kg and the feather has a mass m Kg where $M > m$. Given that the air resistance felt by the cannon ball is negligible and the air resistance felt by the feather has magnitude mkv^2 where $k > 0$ is some constant and v is the speed of the feather, calculate the speed with which the cannon ball and the feather will hit the floor.

9. A vase initially contains 3 black balls and 7 white balls. Apart from colour, the balls are identical. Balls are taken from the vase without looking, so the choice is random.

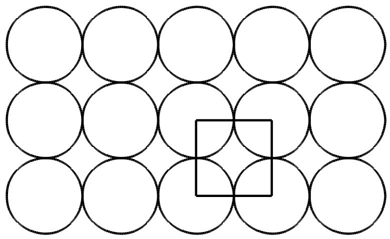
(a) A ball is taken from the vase, its colour is noted, and it is returned. The vase is shaken to mix the balls. Then another ball is taken from the vase. What is the probability that both balls were white? What is the probability that the balls were different colours?

(b) A ball is taken from the vase, its colour is noted, and it is **not** returned, leaving 9 balls in the vase. The vase is shaken to mix the balls. Then another ball is taken from the vase. What is the probability that both balls were white? What is the probability that the balls were different colours?

(c) A ball is taken from the vase, its colour is noted, and three balls of that colour are put in the vase (so the vase now has a total of 12 balls). The vase is shaken to mix the balls. Then another ball is taken from the vase. What is the probability that both balls taken from the vase were white? What is the probability that the balls were different colours?

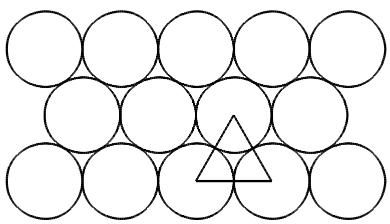
10. We consider two ways for densely packing circles, all of radius 1, into the xy plane. Imagine that the interiors of the circles are shaded, and we would like to cover as much of the area of the xy plane with shaded circles as possible. We shall look at two cases:

The first is a lattice where the circles are stacked horizontally and vertically, so that they touch. This looks like the following picture.



Another way to think of this is as a tiling of square tiles, where one square tile is shown in the picture. Its corners are at the centres of the circles. Find what proportion of the area of the xy plane is covered by circles. To do this, it is sufficient to find what proportion of the square shown is covered by circles.

The second is a lattice where the horizontal rows of circles are staggered, so that they fit into the gaps on the row below, resulting in a picture of touching circles as shown below.



Another way to think of this is as a tiling of equilateral triangular tiles, where one equilateral triangular tile is shown in the picture. Its corners are at the centres of the circles. Find what proportion of the area of the xy plane is covered by circles. To do this, it is sufficient to find what proportion of the equilateral triangle shown is covered by circles.