Hexadecimal Numbers

Hexadecimal (also **base 16**, or **hex**) is a positional numeral system with a *base*, or *radix*, of 16. It uses sixteen distinct symbols, most often the symbols

- **0–9** to represent values zero to nine, and
- **A**, **B**, **C**, **D**, **E**, **F** to represent values ten to fifteen.

For example, the hexadecimal number **A5F3** is equal, in decimal, to

$$(10 \times 16^3) + (5 \times 16^2) + (15 \times 16^1) + (3 \times 16^0),$$

which is

$$(10 \times 4096) + (5 \times 256) + (15 \times 16) + (3 \times 1),$$

and is

$$42{,}403.$$

Hexadecimal is commonly used as a human friendly way to represent computer memory addresses. Hexadecimal provides a concise way of representing long binary numbers. For example, the values of an 8-bit byte can range from

$$00000000,\ 00000001,\ \ldots,\ 11111111$$

i.e., 0 to 255 in decimal. But these 256 numbers may be more conveniently represented by two hexadecimal digits in the range

$$00,\ 01,\ \ldots,\ FF.$$

The advantage of using hexadecimal rather than decimal increases rapidly with the size of the binary number.

When the binary number becomes large, conversion from binary to decimal is slow. However, it is trivial to map between binary and hexadecimal. Each

hexadecimal digit represents four binary digits and we may view the binary number as a string of 4-digits and map each to a single hexadecimal digit independently.

Consider a 16-bit example that compares the conversion of a binary number to decimal and to hex.

Binary to decimal

$$\begin{aligned} 1110\ 1011\ 0101\ 0010 &= 2^{15}+ 2^{14}+ 2^{13}+ 2^{11}+ 2^{9}+ 2^{8}+ 2^{6}+ 2^{4}+ 2^{1} \\ &= 32768 + 16384 + 8192 + 2048 +512+256 + 64 + 16 + 2 \\ &= 60{,}242 \end{aligned}$$

Binary to hex

Here each of the four 4-digit strings can be considered independently, and converted directly:

$$1110\ 1011\ 0101\ 0010 = \text{E B 5 2}$$