Powers of 2

n	2^n	n	2^n	n	2^n	n	2^n
1	2	13	8192	25	33554432	37	137438953472
2	4	14	16384	26	67108864	38	274877906944
3	8	15	32768	27	134217728	39	549755813888
4	16	16	65536	28	268435456	40	1099511627776
5	32	17	131072	29	536870912	41	2199023255552
6	64	18	262144	30	1073741824	42	4398046511104
7	128	19	524288	31	2147483648	43	8796093022208
8	256	20	1048576	32	4294967296	44	17592186044416
9	512	21	2097152	33	8589934592	45	35184372088832
10	1024	22	4194304	34	17179869184	46	70368744177664
11	2048	23	8388608	35	34359738368	47	140737488355328
12	4096	24	16777216	36	68719476736	48	281474976710656

Now $\log(2^n) = n\log(2) \approx 0.301n$, so 2^n has approximately $1 + 0.301n$ digits. Did you know that if you fold a piece of paper 42 times, its thickness will be greater than the distance to the moon? Of course, you may have difficulty doing it after the seventh or eight fold. Each time the paper is folded, the thickness doubles. Thus after 42 folds, the thickness will have been doubled 42 times - that is multiplied by 2^{42}. Now $2^{42} = 4398046511104 \approx 4.39805 \times 10^{12}$.
A ream of A4 paper, 500 pages, is about 5cm thick, so a sheet of paper is about 0.01cm thick. So after folding 42 times, the thickness is $439,805$km. The distance to the moon is only about $400,000$ km.
Mathematicians have proved various results about very large numbers, even though the numbers could never be handled by a computer. One such result is that $2^{2^{73}} + 1$ is not a prime, since it has the factor $5 \times 2^{75} + 1 = 188894659314785808547841$. To write $2^{2^{73}} + 1$ in decimal notation, it would need more than 2^{71} digits since

$$\log(2^{2^{73}} + 1) \approx \log(2^{2^{73}}) \approx 2^{73}\log(2) \approx 2^{71}.$$

Now since $2^{10} \approx 10^3$, we know that

$$2^{71} = 2 \times (2^{10})^7 \approx 2 \times 10^{21}$$

so that $2^{2^{73}} + 1$ has more than 2×10^{21} digits.

Some things to think about.

(1) Find out the earth's circumference in kilometres, then its radius, and its surface area.
(2) If we wrote each digit in a square 0.2 cm wide, how many digits would cover the earth's surface?
(3) Use a computer to calculate the approximate value of

$$\log(1) + \log(2) + \cdots + \log(100).$$

Approximately how many digits does 100! have?
(4) Approximately how many ways can I choose 50 things from 100?
(5) Approximately how many ways can I split up 100 things into 50 pairs?
(6) Approximately how many digits does $2^{2^{100}}$ have?
(7) What is the smallest integer n for which 2^n has at least 100 digits?
(8) Look at the last digit of 2^n. Can you see a pattern? Can you explain it? What is the last digit of 2^{1000}?
(9) What are the last two digits of 2^{1000}?

Department of Mathematics
SWANSEA UNIVERSITY
PRIFYSGOL ABERTAWE
email: mathematics@swansea.ac.uk
http://www-maths.swan.ac.uk